

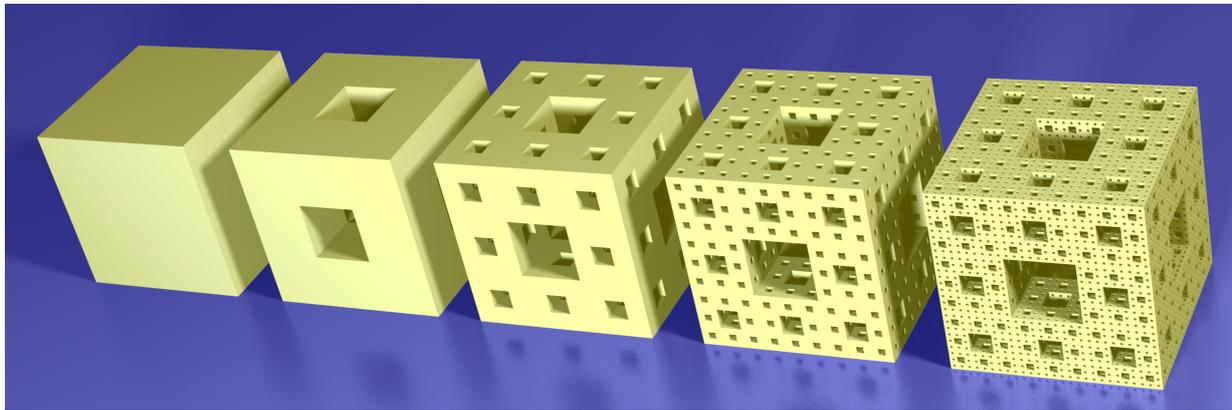


3D Fractals

There are lots of different three-dimensional fractals, many of which can be made using a repeated process.

Menger Sponge

A Menger Sponge is the three-dimensional version of the Sierpinski Carpet - it's made by splitting a cube into 27 smaller cubes, and removing 7 of them leaving a frame with a hole in each face. This is then repeated for each of the remaining smaller cubes, and so on.

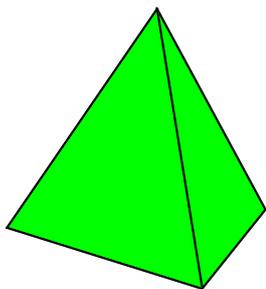


Construction of a Menger Sponge: based on an image by Niabot, CC-BY 3.0, via Wikimedia Commons

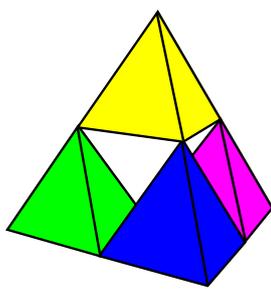
Sierpinski Tetrahedron

A Sierpinski Tetrahedron, sometimes also called a Tetrix, is a three-dimensional version of the Sierpinski triangle.

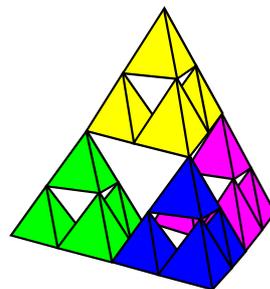
It's made by removing the middle section of a tetrahedron, leaving four smaller ones, and then repeating forever - or, alternatively, you can think of it as being made by joining together four smaller tetrahedra, then joining four of the larger ones, and so on.



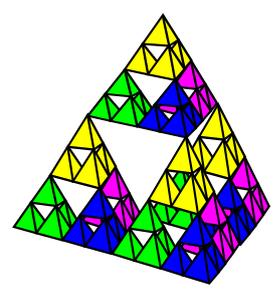
Stage 1 (tetrahedron)



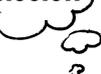
Stage 2



Stage 3



Stage 4

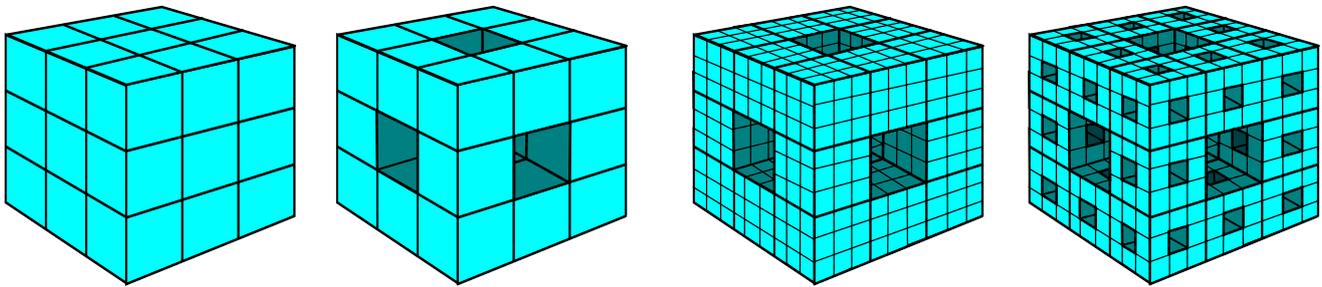




Questions

1. For the Menger Sponge:

- How many small cubes are there in the first stage, after you remove one set of 7 cubes? If you split each of these cubes into smaller cubes, and remove 7 from each to make the stage 2 Menger Sponge, how many of these smaller cubes do you have?
- If the whole cube measured 27cm along the edge, what is the length of the edge of one of the small cubes making up a stage 3 sponge?
- BONUS: What fraction of the volume of the whole cube is removed to go from Stage 1 to Stage 2? (Hint: It's not $7/27$).



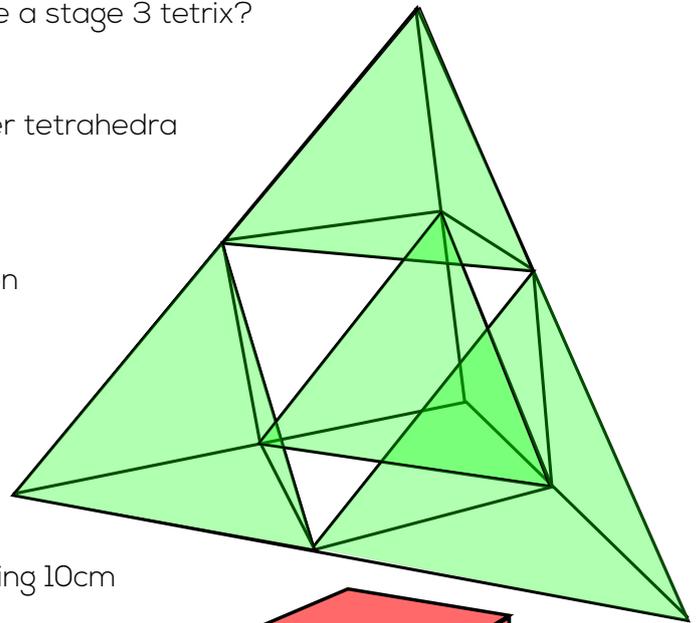
2. For the Sierpinski tetrahedron:

- How many smaller tetrahedra are needed to make a stage 3 tetrix?
- What about a stage 4 tetrix?
- Is there an easy way to calculate how many smaller tetrahedra are needed to make any stage?

Bonus question:

If you wanted to balance a Sierpinski Tetrahedron on top of a Menger Sponge, which means the face of the Menger Sponge needs to be bigger than the hole in the bottom of the Sierpinski Tetrahedron, and your Menger Sponge has side length 21 centimetres, what does the side of your tetrahedron need to measure?

If you're building it from smaller tetrahedra measuring 10cm along each edge, how many will you need?



Further Research:

- Who were Cantor, Sierpinski, Menger, and Koch? What can you find out about them?